Challenges in Leakage-Resilient Symmetric Cryptography

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Provable Security

Define "Breaking the Cryptosystem".

- 2 Construct Cryptosystem.
- Prove Cryptosystem Secure.





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Define "Breaking the Cryptosystem".
 Example: Digital Signatures





 ${f
m I}$ breaks scheme if ${f
m P}$ is a valid signature for a new message.

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Provable Security

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 Example: Digital Signatures





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Theorem

No efficient adversary who breaks the scheme exists





lacksquare breaks scheme if $ar{B}$ is a valid signature for a new message.

- Onstruct Cryptosystem.
- Prove Cryptosystem Secure.

Theorem

No efficient adversary who breaks the scheme exists if (factoring, SVP,...) is hard.

• Provably secure cryptosystems get broken in practice.

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- Problem: adversaries outside the anticipated model.



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• E.g. can measure time to compute 🤛



- E.g. can measure time to compute
- breaks RSA on smart cards [Kocher'95]



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Side-Channel Attack: Cryptanalytic attack exploring information leaked from a physical implementation of a cryptosystem. • power analysis



• probing attacks

cold-boot attacks

cache attacks







• radiation, sound, heat,...



- power analysis
 [Eisenbarth et al. CRYPTO'08]
 break wireless car keys
- probing attacks





- cold-boot attacks [Halderman et al. USENIX'08] break disc-encryption schemes
- cache attacks [Ristenpart et al. CCS'09] break cloud computing
- radiation, sound, heat,...











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 - Ubiquitous computing: Light-weight crypto-devices are susceptible to side-channel attacks.
 - Provable security: Side-channels became the weakest link.



Side-channels are a physical phenomenon, how could theoretical cryptography be of help?



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- Reductions in the context of side-channel attacks [MicRey'04]
- Construct schemes that remain provably secure in the presence of leakage.



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Leakage models: one-time vs. continuous



- Most side-channels like timing, power, . . . are continuous. Notable exception cold-boot.
- Security against continuous leakage is *much* harder to achieve. E.g. requires key-refreshing.
- Intermediate "Floppy model".

Leakage models: dedicated vs. general

dedicated leakage functions

f models a particular side-channel timing: Make running time independent of input. probing: Private Circuits ([Ishai,Sahai,Wagner Crypto'03])

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general leakage functions

bounded: f(key) has length $\ell \ll |key|$ bits.

entropic: Entropy of *key* decreases by at most ℓ given f(key).

auxiliary input: Computationally hard to compute key given f(key).

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One-Time Bounded/Entropic leakage

 $key \in \{0,1\}^n$. Adv choses f and gets f(key).

- **O** Bounded leakage: f must satisfy $|f(key)| = \ell \ll n$.
- 2 Entropic leakage: f must satisfy $H_{\infty}(key|f(key)) \ge n \ell$.
 - Maurer's bounded storage model, privacy amplification,...
 - Intrusion resilience [Dzi'06,CDDLLW'07,...] (symmetric)
 - Memory attacks [AGV'09,NaoSeg'09,...] (public-key)

- *key*_i state after *i*'th invocation of the scheme.
- $key_i^+ \subseteq key_i$ touched in *i*'th invocation.

Before *i*'th invocation, Adv chooses f(.) with range $\{0, 1\}^{\ell}$ and gets

- $f(key_i^+)$ (Leakage-Resilient Cryptography [DziPie08],...)
- f(key_i) (Continuous Memory Attacks [DHLW12],...)

Public-key

Signatures: [AWD09, KV09, FKPR10, DHLW10, BKKV10, BSW11,...] Public key encryption: [AGV09, NS09, DHLW10, BKKV10, BSW11,...] Identity based encryption: [DHLW10, CDRW10, LRW11,...] Multiparty Computation: [FRRTV10, GR10, JV10,...] Zero Knowledge: [GJS11,...]

Secret-key

Stream-Ciphers: [DP08, Pie09, YSPY10, YS12,...]

Pseudorandom Functions/Permutations: [DP10, FPS11, MSJ12,...]

Compilers

[ISW03,FRRTV10,GolRot12,...]

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3 Principles

- Share Secret: Blinding
- Evolve Secret: Stream-Ciphers
- Hide Secret: For every *pk* many *sk* (HPS,Σ-Protocols)

Leakage-Resilient Stream-Ciphers

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weak PRFs

Definition Weak PRF

 $\mathcal{F}: \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^m \text{ is a } (s,\epsilon,q) \text{ secure weak PRF}$ if no adversary of size s can distinguish the following distributions advantage ϵ

$$(X_1, Y_1), \ldots, (X_q, Y_q)$$
 $(X_1, Z_1), \ldots, (X_q, Z_q)$

where X_i, Z_i are uniform and $Y_i = \mathcal{F}(K, X_i)$ for a random K.

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where X_i, Z_i are uniform and $Y_i = \mathcal{F}(K, X_i)$ for a random K.

Definition Min-Entropy

X has min entropy m if $\Pr[X = x] \le 2^{-m}$ for all x.

If $K \in \{0,1\}^k$ is uniform and $|f(K)| = \lambda$, then K has $k - \lambda$ bits min-entropy given f(K).

Definition (Stream-Cipher)

• A function $\mathcal{SC} : \{0,1\}^{\kappa} \to \{0,1\}^{n} \times \{0,1\}^{\kappa}$ is a stream-cipher if for random K_{0} the output Y_{1}, Y_{2}, \ldots (where $(K_{i}, Y_{i}) = \mathcal{SC}(K_{i-1})$) is pseudorandom

$$\begin{array}{c} K_0 \rightarrow \underbrace{\mathcal{SC}}_{V_1} \longrightarrow \underbrace{\mathcal{SC}}_{V_2} \longrightarrow \underbrace{\mathcal{SC}}_{V_3} \longrightarrow \underbrace{\mathcal{SC}}_{V_4} \longrightarrow \underbrace{\mathcal{SC}}_{$$

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$$\begin{array}{c} K_0 \longrightarrow F \longrightarrow K_1 \longrightarrow F \longrightarrow K_2 \longrightarrow F \longrightarrow K_3 \longrightarrow F \longrightarrow K_4 \\ \Lambda_1 & Y_1 & \Lambda_2 & Y_2 & \Lambda_3 & Y_3 & \Lambda_4 & Y_4 \end{array}$$

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Can use any pseudorandom generator F

But not leakage resilient even for $\lambda = 1$: For $t = |\mathbf{K}| + 1$, define

$$\Lambda_i = f(K_{i-1}) \stackrel{\text{def}}{=} i'th \ bit \ of \ K_t.$$

After t rounds leaked entire K_t .





 $\Lambda_1 = f(\mathbf{K_0}, \mathbf{Y_0})$





 $\Lambda_2 = f(\mathbf{K_1}, \mathbf{Y_1})$

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Theorem ([P'09])

If F is a wPRF then the above is a leakage-resilient stream-cipher: Given Y_0, \ldots, Y_i and $\Lambda_1, \ldots, \Lambda_i$ the Y_{i+1}, Y_{i+2}, \ldots is pseudorandom.

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• Leakage function $f(K_i, Y_i) \rightarrow \Lambda_i$ can't compute K_{i+2}, K_{i+3}, \ldots

Quantitative bound in [P'09] is nowhere practical.

 (s, ϵ) secure wPRF gave (s', ϵ') secure stream cipher where

$$\epsilon' \approx \epsilon^{1/12} \qquad s' \approx s \cdot \epsilon^2$$

As $\log(s/\epsilon) \leq$ key length, require wPRF with key length \gg 1000 to get meaningful bounds.

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With two recent results we can give a meaningful bound for keys of length 256.

- Overcoming weak expectations. [DodisYu 2012]
- How to fake auxiliary input. [JetchevP 2012]

Overcoming Weak Expectations

Theorem [DodisYu 2012] (improving [P'09])

If \mathcal{F} is a $(\epsilon, 2s, 2q)$ secure wPRF, then it is a

$$(\sqrt{2^{\lambda}\epsilon}, s, q)$$

secure wPRF if the key $K \in \{0, 1\}^k$ comes from any distribution with $k - \lambda$ bits of min-entropy.

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secure wPRF if the key $K \in \{0,1\}^k$ comes from any distribution with $k - \lambda$ bits of min-entropy.

Every weak PRF is one-time bounded leakage-resilient!

Every also holds for entropic leakage (if leakage function is efficient).

Theorem [JetchevP 2012]

• Consider any joint distribution $(X, A) \in \mathcal{X} \times \{0, 1\}^{\lambda}$.

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Theorem [JetchevP 2012]

- Consider any joint distribution (X, A) ∈ X × {0,1}^λ.
- Let \mathcal{D} be a class distinguishers, say circuits of size $s = 2^{80}$
- There exists an efficient simulator h : X → {0,1}^λ that
 Fools every D in D

 $\forall D \in \mathcal{D} : |\Pr[D(X, A) = 1] - \Pr[D(X, h(X)) = 1]| \le \epsilon$

• *h* is of size $s \cdot 2^{3\lambda} / \epsilon^2$.



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• Replace Λ_1 with "fake" $h(K_2, Y_1)$.



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 λ : # of bits leaked per round. q : # of blocks output.

Lemma ([JetPie'12])

If F is a $(\epsilon_{F}, s_{F}, 2)$ -secure weak PRF the this is a $(\epsilon', s', q, \lambda)$ -secure leakage resilient stream cipher where

$$\epsilon' = 4q\sqrt{\epsilon_{\mathsf{F}}2^{\lambda}} \qquad s' = \Theta(1)\cdot rac{s_{\mathsf{F}}\epsilon'^2}{2^{3\lambda}}$$

$$q = 2^{20} , \ \lambda = 10 , \ \epsilon_{\mathsf{F}} = 2^{-100} , \ s_{\mathsf{F}} = 2^{154} \quad (s_{\mathsf{F}}/\epsilon_{\mathsf{F}} = 2^{256})$$

 $\epsilon' = 2^{-23} \qquad s' = 2^{78}$

Leakage-Resilient PRFs

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GGM



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GGM + LR-SC



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$GGM + LR-SC \Rightarrow LR-PRF$



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- Granular Leakage.
- On-adaptive leakage.
- 1. & 2. allow static key!
- Inefficient construction.

Leakage-Resilient PRPs

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Theorem ([LubyRackoff'88])

3-round Feistel instantiated with PRFs is a PRP.



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18-round Feistel instantiated with URFs is indifferentiable from a URP.



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Theorem ([DodisP'10])

r-round Feistel instantiated with leakage-resilient PRFs is a secure leakage-resilient super PRP for q-query distinguishers satisfying $q \leq 1.38^{r/2-1}$.

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Side-Channel Attacks on Feistel



 Ψ_r : *r*-round Feistel instantiated with uniformly random functions $\{0,1\}^n \to \{0,1\}^n$.

Theorem ([DodisP'10])

Can invert Ψ_r on any value Y making $4n^r$ forward queries. If given $|Z_1|_1, \ldots, |Z_n|_1$ with every query.

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• Works for other leakages (than Hamming weight) of the Z_i's.

getting LR PRFs is hard what to do? Use algebraic PRFs, e.g. f(x) = g<sup>a₀ | 1_{x_i=1} a_i
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 [NaorReingold'97]. Can use blinding to protect.
 </sup>
- Avoid PRFs! Use algebraic MACs [DodKilPieWic'12] like
 LaPiN



A Proposal: LaPiN [HeyKilLyuPaaP FSE'12]

$$\frac{\text{Ring}^{1} \quad \text{R} = \mathcal{F}_{2}[X]/(f)}{\underbrace{\frac{P \text{rover}}{\leftarrow}} \qquad \underbrace{\frac{V \text{erifier}}{}}_{\text{Random challenge } c \in \{0,1\}^{80}}$$

$$c \text{hose } r, e \in \mathbb{R}$$

$$z = r \cdot (k \cdot \pi(c) + \hat{k}) + e \in \mathbb{R} \xrightarrow{r,z} \hat{e} = z - r \cdot (k \cdot \pi(c) + \hat{k})$$
Accept if \hat{e} is a small element in ring R.

• Key are two ring elements k, \hat{k} (621 bits each)

• Share
$$k = k_0 \cdot k_1, \hat{k} = \hat{k}_0 \cdot \hat{k}_1$$

- Run protocol using (k_i, k̂_i) for i ∈ {0,1}, combine at the end.
- Occasionally refresh $k_0 \leftarrow k_0 \cdot z$, $k_1 \leftarrow k_1 \cdot z^{-1}$.

 $\frac{1}{1} f(X) = (X^{127} + X^8 + X^7 + X^3 + 1)(X^{126} + X^9 + X^6 + X^5 + 1)(X^{125} + X^9 + X^7 + X^4 + 1)(X^{122} + X^7 + X^4 + X^3 + 1)(X^{121} + X^8 + X^5 + X^1 + 1) + 1)$

Auxiliary Input Security?

Auxiliary Input vs Bounded Leakage: A Conjecture

Adversary gets Bounded Leakage: f(key), $|f(key)| \le \ell$.

Auxiliary Input: f(key), key is hard to compute given f(key).

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Is Aux. Input really stronger than bounded leakage in practice?

Does there exist a *natural* scheme that is secure against bounded leakage, but not auxiliary input (which does not trivially contradict the bounded leakage bound)?

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Does there exist a *natural* scheme that is secure against bounded leakage, but not auxiliary input (which does not trivially contradict the bounded leakage bound)?

RO analogy

Does there exist a *natural* scheme that is secure in the random oracle model, but not if the RO is replaced with, say SHA3.

Questions?



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